Model Order Reduction Techniques for Electric Machine Modeling

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I. Motivation

In times where the world becomes more and more complex so do simulations. Solving models analytically is only possible for sufficient small input which only covers the solutions of real applications. As soon as more complicated models are used the computational complexity is the limiting factor. It can be either the required memory or the needed time to solve such models, both not acceptable to an alternative. Yet, to solve mathematical models of numerical simulations to a satisfying level one method yields promising results: model order reduction (MOR). The goal of MOR is to approximate the input-to-output behaviour $\|y - \hat{y}\| \ll 1$ while reducint the number of system states and differential equations $r \ll n$

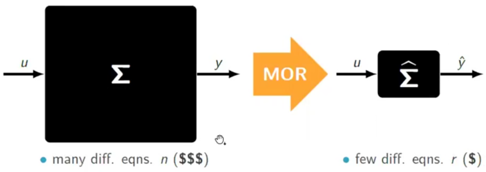

Fig. 1.1: Visualisation of basic concept of MOR. A large system Σ with described with many equations n can be reduced to a smaller system $\hat{Sigma}$ with many fewer equations r.

II. Mathematical background

Mathematical models are often described with partial differential equations (PDE) and a common way to describe the behaviour of an input-output system is given by

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \qquad \text{[II..1]}$$

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \qquad \text{[II..2]}$$

with $\boldsymbol{u}$ as the system input, $\boldsymbol{y}$ the system output and $\boldsymbol{x}$ the state variable. The higher the dimension n of the state space vecotr $\boldsymbol{x}$, the higher the complexity. MOR is the reduction of the dimension of the state vector by keeping the behaviour of the input-output relations.

Proper Orthogonal Decomposition (POD) is based on the assumption that a limited number of deterministic function, the POD modes, are sufficient to predict future behaviour. If we have a vector-valued function $u(\boldsymbol{x}, t)$ over some domain of interest and time, we can express the quantity of interest with the standard eigenfunction expansion

$$\boldsymbol{u}'(\boldsymbol{x}, t) = \sum_{k=1}^{\infty} a_k(t)\Phi_k(\boldsymbol{x}) \qquad \text{[II..3]}$$

For each $\boldsymbol{x}$ spatial value and t time value we can create the so-called snapshot matrix U from which the eigenvalues and eigenvectors can be computated.

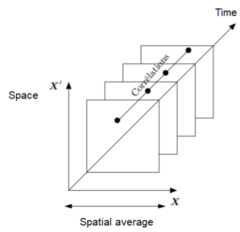

Fig. 2.1: Visualization of snapshot matrix, each new *snapshot* leads to a new entry in the snapshot matrix U as a column whereas the rows hold the values of space.

It turns out that not all eigenvectors are needed to reconstruct the behaviour and therefore a reduction of the original system can be performed. The more snapshots are into taken account, the more data is provided and the better the time evolution of the system can be inspected.

III. Geometry setup

To evaluate theory on a model the following model of a motor was used and the quantity of interest was the magnetic flux density Φ. The numerical computation was done with COMSOL 6.2 and the data evaluation with Python in its latest version 3.12.3.

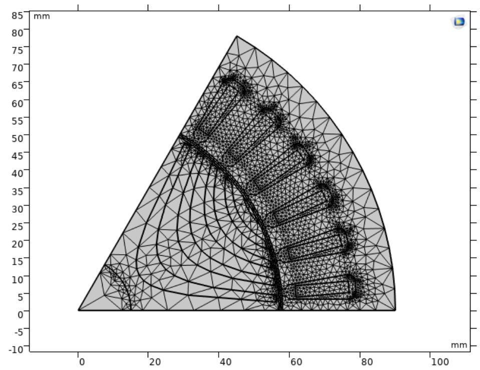

Fig. 3.1: 2d electrical motor model that consists of a stator, rotor and its windings.

The model consists of a 6.0° degree section of a motor with the stator as the fixed and the rotor as the moving part. The total extension of the motor is 9.0×10^1 mm with 5.75×10^1 mm as the rotor dimension. For numerical simplification the windings, air gap were simulated as air whereas everything else was modeled in soft iron without losses. Periodic boundary conditions were applied for the rotor and motor part and the air gap as well as outer part of the stator are kept magnetic isolated. The original degrees of freedom to solve in the setup was 3.2300×10^4, in total there were 6.7×10^1 different snapshots taken and the reduced matrix contains 9.379×10^3 rows.

IV. Results

The following table summarizes the results of the simulation and one can already see that the first mode dominate the system by storing more than 90 % of the total energy.

POD	eigenval	acc part E
1	2.034348e+05	0.909562
2	1.954733e+04	0.996958
3	6.376754e+02	0.999809
4	3.270508e+01	0.999956

Tab. 4.0: Only 2 modes are already sufficient to cover more than 99 % of the energy stored in the system.

One can also compare the simulated values of the magnetic flux density with the reconstructed values based on the POD modes.

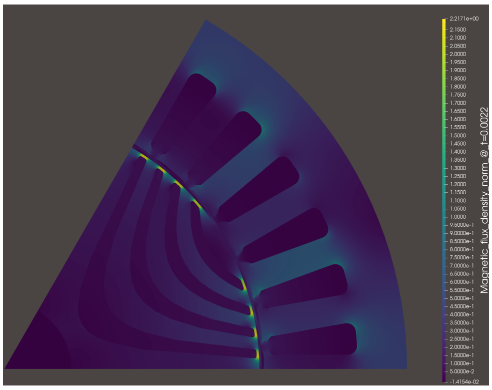

Fig. 4.1: Reconstructd based on POD.

As one can see, the generated valus by the POD modes do not only qualitatively but also quantitatively describe the systems behaviour. The rotation of the rotor part in the simulation can be neglected for the explanation.

V. Outlook

The current results show that the implemented linear POD is in agreement with the unreduced model, so for further study the next step arise naturally on focusing on nonlinear method. Typical cases for nonlinear situations is the extension of the nonlinear magnetization dynamic of magnetic material as in BH curve.

Acknowledgement

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